choice of $R$, a different $\tilde{Q}_l$ would result to achieve the same objectives. The matrix $Q_l$, the corresponding gain matrix $K_l$ and $F_l = F - GK_l$ were determined.

Thus far the two complex poles have been moved to more desirable locations and the zero movement determined as shown in Fig. 1b. The feedback gains at this stage shifted the open-loop unstable pole to its mirror image in the left half-plane. This is the property of optimal control design. The stable open-loop pole has remained at the same location. The zeros of the transfer function of the pitch attitude to pitch control commands are affected by feedback from the second input, the thrust control command. The elements of the feedback gain matrix corresponding to the feedback from the second input are small and have not moved the zeros of $\theta/\delta_{es}$ significantly from their open-loop values, but the zeros of $\theta/\delta_t$ have moved closer to the real poles.

In the next step, one of the real poles is moved close to one of the zeros of $\theta/\delta_{es}$ and the zeros of $\theta/\delta_t$ transfer functions moved close to the real poles. To do this, $F_l$ was transformed into block diagonal form and $\tilde{Q}_2$ was selected so that only the real pole closer to the origin and zeros of $\theta/\delta_t$ were moved close to the zeros of $\theta/\delta_{es}$ transfer functions as shown in Fig. 1c. The matrices $Q_2$ and $K_2$ were computed from $\tilde{Q}_2$. The final closed-loop matrix was computed by $F_c = F_l - GK_2$, and the final performance index matrix $Q = Q_l + Q_2$. It is noted that with sufficiently high feedback gains, one of the real poles and the zeros of $\theta/\delta_t$ transfer function are driven into close proximity to the zeros of the $\theta/\delta_{es}$ transfer function, so that the aperiodic pair is close to the zeros of $\theta/\delta_{es}$ and $\theta/\delta_t$ transfer functions, and the attitude transfer functions are essentially second order.

The numerator zeros of $\theta/\delta_{es}$ have moved very little from their open-loop locations. Figure 1c shows the final pole-zero locations and the two real poles approximately cancel the zeros of the $\theta/\delta_{es}$ and $\theta/\delta_t$ transfer functions. Thus, the attitude response is essentially second-order dominated by the modified short-period mode. The responses of pitch attitude and pitch rate to a step input are shown in Fig. 1d. The time history illustrates the second-order nature of attitude response. The design example presented in this section has demonstrated the usefulness of the step-by-step approach to control system design. The sequential procedure provides the designer tractable information to select the performance index matrices.

## Conclusions

Two sequential design procedures were developed for optimal control system design. These procedures determine the pole-zero movements at each stage as the weighting matrix of the performance index is varied. The weighting matrices constructed at each stage were added to get the final weighting matrix to move the open-loop poles and zeros to more desirable locations. The second design procedure based on the eigenvector approach is considered to be more promising because of increased tractability of the modified system at every stage of the design process, and is being further developed into a systematic design procedure by the authors.

## Acknowledgment

This work was supported by the Office of Naval Research under Contract No. N00014-78-C-0155.

## References

[1] Anderson, B.D.O. and Moore, J.B., *Linear Optimal Control*, Prentice-Hall, Englewood Cliffs, N.J., 1971.

[2] Rynaski, E.G. and Whitbeck, R.F., "The Theory and Application of Linear Optimal Control," Calspan Corporation, Buffalo, N.Y., Calspan Rept. No. IH-1943-F-1, Oct. 1965.

[3] Solheim, O.A., "Design of Optimal Control Systems with Prescribed Eigenvalues," *International Journal of Control*, Vol. 15, 1972, pp. 143-160.

[4] Whitbeck, R.F., "A Comment on the Linear Optimal Control Problem," *IEEE Transactions on Automatic Control*, Vol. AC-12, Dec. 1967, pp. 777-778.

[5] Govindaraj, K.S., Rynaski, E.G., and Fam, A.T., "Design Criteria for Optimal Flight Control Systems," Calspan Corporation, Buffalo, N.Y., Calspan Rept. No. 6248-F-1, Sept. 1979.

G80-065

# Optimal Member Damper Controller Design for Large Space Structures

30003
70004

Suresh M. Joshi*
*Old Dominion University Research Foundation, Norfolk, Va.*
and
Nelson J. Groom†
*NASA Langley Research Center, Hampton, Va.*

## Introduction

CONTROL systems design for large flexible space structures is a complex and challenging problem because of their special dynamic characteristics. Large space structures tend to have extremely low-frequency, lightly damped bending modes which are closely spaced in the frequency domain. Because of pointing requirements, a number of lower frequency modes will probably fall within the bandwidth of the primary controller, thus making active control of some of the modes unavoidable. Since control of all the modes is impractical, the primary controller will be reduced-order. This introduces stability problems because of observation and control spillover.[1-3] The stability of a system with a reduced-order controller is heavily dependent on the natural damping of the residual (uncontrolled) modes. Therefore, it is desirable to increase the damping of the residual modes where possible.

The member damper approach and the application of multiple-member dampers in an output velocity feedback configuration was discussed in Ref. 4. The member damper approach includes local damping elements which could consist of colocated actuators and velocity sensors. Each actuator sensor pair is configured as a single-loop control system and the member dampers work independently of each other. In the output velocity feedback configuration, all the sensor signals are distributed by a gain matrix to interconnect all the actuators and sensors. This concept was further investigated in Refs. 5 and 6. It has been proved in these references that direct velocity feedback (DVFB) with colocated sensors and actuators cannot destabilize the system. Such controllers may be used in conjunction with a conventional (modern) active controller, and have the potential to effect significant improvement in the overall performance.

Selection of velocity feedback gains for individual member dampers is an important part of the design. The root locus technique may be used for this purpose; however, this could be a complex task, especially if a large number of actuators are used. In this note, the problem of selecting velocity feedback gains is formulated as an optimal output feedback regulator problem, and the necessary conditions are derived for minimizing a quadratic performance function. The special structure of the gain matrix (i.e., diagonal) is taken into account, and the knowledge of process noise and sensor noise is used to advantage.

Received Nov. 6, 1979; revision received Jan. 14, 1980. 

Index categories: Spacecraft Dynamics and Control; Analysis and Numerical Methods.

*Associate Professor (Research).
†Aero-Space Technologist. Member AIAA.

## Equations of Member Damper Controllers

The structural model of a large space structure can be (approximately) described by the equations

$$\ddot{q}_0 + D_0\dot{q}_0 + \Lambda_0 q_0 = \Phi_0^T f \tag{1}$$

$$y_0 = \Phi_0 q_0 \tag{2}$$

where $q_0$ is the $n_0$-dimensional vector of modal amplitudes, $\Phi_0$ is the $m \times n_0$ "mode shape" matrix, and $f$ is the $m \times 1$ generalized force vector [components of $f$ represent applied forces (or torques)], $y_0$ is the $m \times 1$ vector of generalized displacements (linear and angular) at the $m$ points of application $(l_1, l_2, \ldots l_m)$ of the generalized forces, $D_0$ is the inherent damping matrix, and $\Lambda_0$ is the diagonal matrix of squared natural frequencies. These equations describe truncated normal-coordinate continuous models, or finite-element models.

If a member damper is connected between two points, equal and opposite forces (torques), proportional to the sensed relative velocity (relative angular velocity) between the points, are applied at the points. Thus, if a single member damper is connected between points $l_1$ and $l_2$, the equations are

$$\ddot{q}_0 + D_0\dot{q}_0 + \Lambda_0 q_0 = [\phi_1 \phi_2] \begin{bmatrix} f \\ -f \end{bmatrix} = [\phi_1 - \phi_2] f \tag{3}$$

where the $n_0$-vector $\phi_i$ is the $i$th column of the $\Phi_0^T$ matrix, and $f$ is the scalar force:

$$f = g_1 \dot{y}_1 \tag{4}$$

$$y_1 = (\phi_1^T - \phi_2^T) q_0 \tag{5}$$

where $g_1$ is the DVFB gain ($g_1$ and $y_1$ are scalars).

Substitution of Eqs. (4) and (5) into Eq. (3) yields

$$\ddot{q}_0 + (D_0 - g_1 \psi_1 \psi_1^T)\dot{q}_0 + \Lambda_0 q_0 = 0 \tag{6}$$

where

$$\psi_1 = \phi_1 - \phi_2 \tag{7}$$

If $p$ member dampers are used, the closed-loop equation becomes

$$\ddot{q}_0 + \left(D_0 - \sum_{i=1}^{p} g_i \psi_i \psi_i^T\right)\dot{q}_0 + \Lambda_0 q_0 = 0 \tag{8}$$

where $g_i$ is the feedback gain and $\psi_i$ the effective input matrix [similar to Eq. (7)] for the $i$th damper. If $g_i \leq 0$ $(i = 1,2,\ldots,p)$ and $D_0 \geq 0$, then the effective damping matrix [coefficient matrix of $\dot{q}_0$ in Eq. (8)] is positive semidefinite, and the system is stable in the sense of Lyapunov; if the effective damping matrix is positive definite, the system is asymptotically stable.[5] It should be noted that this is only a sufficient condition, and the system can be asymptotically stable even though the effective damping matrix is only positive semidefinite.

## Optimal Output Feedback Formulation

For the purpose of controller design, $n$ of the $n_0$ modes of the structure are considered. Thus, the "design model" is of order $2n$. Although the member damper control system is based upon the lower order "design model," it cannot destabilize the higher order model. Therefore, this design does not suffer from the problem associated with the use of reduced-order models in conventional optimal regulator and estimator design. Let $q$ denote the modal amplitude vector for the modes in the "design" model. The state equations for the system under consideration, including process noise and sensor noise, may be written as

$$\dot{x} = Ax + Bu + v \tag{9}$$

$$z = \psi^T \dot{q} + w = Cx + w \tag{10}$$

where

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}_{2n \times 1} \quad A = \begin{bmatrix} 0 & I \\ -\Lambda & -D \end{bmatrix}_{2n \times 2n} \quad B = \begin{bmatrix} 0 \\ \psi \end{bmatrix}_{2n \times p} \tag{11}$$

$$C = [0 \quad \psi^T]_{p \times 2n} \qquad \psi = [\psi_1 \psi_2 \ldots \psi_p]_{n \times p} \tag{12}$$

$x$ is the $2n \times 1$ state vector, $u$ is the $p \times 1$ vector of effective inputs, $v$ and $w$ are zero-mean, white process noise and measurement noise vectors with covariance intensities $V$ and $W$. It should be noted that $C = B^T$. It is required to obtain an input of the type

$$u = Gz = G(Cx + w) \tag{13}$$

where

$$G = \begin{bmatrix} g_1 & 0 & \ldots & 0 \\ 0 & g_2 & \ldots & 0 \\ 0 & 0 & \ldots & g_p \end{bmatrix} \tag{14}$$

which will minimize

$$J = \lim_{t_f \to \infty} \frac{1}{t_f} \mathcal{E} \int_0^{t_f} \{x^T Q x + (GCx)^T R (GCx)\} \mathrm{d}t \tag{15}$$

where $Q = Q^T \geq 0$, and $R = R^T > 0$ are the state and control weighting matrices, and $\mathcal{E}$ denotes expected value. The noise-dependent part of $u$ is excluded from the performance function $J$ since it makes $J$ unbounded.[7] The optimal output feedback problem for a general nondiagonal rectangular $G$ matrix was solved in Ref. 7. However, in this case, since $G$ is diagonal, the minimization has to be performed with respect to the $p$ variables $g_1, g_2, \ldots, g_p$. (If cross-feedbacks are allowed, the problem becomes the same as the general optimal output feedback problem, with the constraint that the closed-loop system is stable.)

Let the symbol $\alpha * \beta$ denote the element-by-element product (matrix) of matrices $\alpha$ and $\beta$ of the same dimensions. That is,

$$\{\alpha * \beta\}_{ij} = \alpha_{ij} \beta_{ij} \tag{16}$$

Define the $l \times 1$ vector-function $\Delta$ of a $l \times l$ matrix $\alpha$ as

$$\Delta(\alpha) = \begin{bmatrix} \alpha_{11} \\ \alpha_{22} \\ \vdots \\ \alpha_{ll} \end{bmatrix} \tag{17}$$

*Theorem*: The necessary conditions for the minimization of $J$ in Eq. (15), with constraints of Eqs. (9, 10, and 13), are given by

$$g = -[R * (B^T \Sigma B) + W * (B^T P B)]^{-1} \Delta(B^T \Sigma P B) \tag{18}$$

$$(A + BGB^T)^T P + P(A + BGB^T) + Q + BGRGB^T = 0 \tag{19}$$

$$(A + BGB^T)\Sigma + \Sigma(A + BGB^T)^T + V + BGWGB^T = 0 \tag{20}$$

where $P$ and $\Sigma$ are $2n \times 2n$ symmetric matrices, and $g = (g_1, g_2, \ldots, g_p)^T$.

*Outline of Proof*: The structure of the proof is very similar to that used for the general optimal output feedback problem.[7] (It should be noted that the proof given in Ref. 7 needs slight modification in light of Ref. 8, although the end result is the same.) The only difference is that the derivative of the Hamiltonian with respect to the vector $g$ (rather than the matrix $G$) is equated to zero. The following easily proved properties of the matrix trace are used ($G$ is a diagonal matrix, $\alpha$ and $\beta$ are square matrices of compatible dimension).

$$\frac{\partial}{\partial g} \mathrm{tr}[G\alpha\, G\beta] = \frac{\partial}{\partial g}[g^T(\alpha * \beta) g] = \{\alpha * \beta + (\alpha * \beta)^T\} g \tag{21}$$

$$\frac{\partial}{\partial g} \mathrm{tr}[G\alpha] = \Delta(\alpha) \tag{22}$$

The fact that $C = B^T$ is also used.

It should be noted that, as in the case of the general optimal output feedback problem, the theorem does not guarantee the existence of a $g$ that will make the system asymptotically stable, although the necessary conditions assume the existence. Indeed, the performance function of Eq. (15) will be meaningful only if such a $g$ exists.

The optimal gain vector $g$ may be computed using the algorithm given in Ref. 7, or using a numerical minimization method (such as Davidon-Fletcher-Powell). The algorithm of Ref.7 involves iteratively solving Eqs. (18-20). That is, assuming an initial stable $g$, Eqs. (19) and (20) are solved for $P$ and $\Sigma$, and Eq. (18) is solved to obtain the next value of $g$, and so forth. Convergence of the algorithm has not been proven, although it has generally been found to converge[7] in the case of a nondiagonal gain matrix.

## Concluding Remarks

Necessary conditions are obtained for minimizing a quadratic performance function under the framework of the member damper concept. Knowledge of noise covariances is used in the design. The method presented offers a systematic approach to the design of a class of controllers for enhancing structural damping in large space structures. This type of controller has significant potential if used in conjunction with a reduced-order optimal controller that is designed to control rigid-body modes and some selected structural modes.

## References


[1] Balas, M.J., "Feedback Control of Flexible Systems," *IEEE Transactions Automatic Control*, Vol. AC-23, Aug. 1978, pp. 673-679.

[2] Joshi, S.M. and Groom, N.J., "Design of Reduced-Order Controllers for Large Flexible Space Structures," *Proceedings of 16th Annular Allerton Conference on Communication, Control, and Computing*, Oct. 4-6, 1978, pp. 780-791.

[3] Joshi, S.M. and Groom, N.J., "Stability Bounds for the Control of Large Space Structures," *Journal of Guidance and Control*, Vol. 2, July-Aug. 1979, pp. 349-351.

[4] Canavin, J.R., "Control Technology for Large Space Structures," AIAA Paper 78-1691, presented at AIAA Conference on Large Space Platforms, Los Angeles, Calif., Sept. 1978.

[5] Balas, M.J., "Direct Velocity Feedback Control of Large Space Structures," *Journal of Guidance and Control*, Vol. 2, May-June 1979, pp. 252-253.

[6] Strunce, R.R. and Henderson, T.C., "Application of Modern Modal Controller Design to a Large Space Structure," Paper presented at Second VPI&SU Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, Va., June 1979.

[7] Joshi, S.M., "Design of Optimal Partial State Feedback Controllers for Linear Systems in Stochastic Environments," *Proceedings of 1975 IEEE Southeastcon*, Charlotte, N.C., April 1975.

[8] Walsh, P.M., "On Symmetric Matrices and the Matrix Minimum Principle," *IEEE Transactions, Automatic Control*, Vol. AC-22, Dec. 1977, pp. 995-996.


G80-~~[illegible]~~066

# Passive Dissipation of Energy in Large Space Structures

70004
80006


P.C. Hughes*
*University of Toronto, Toronto, Canada*


## Introduction

STRUCTURAL damping models currently used in the dynamical analysis of flexible spacecraft frequently lack fidelity. Yet, an accurate assessment of energy dissipation is often crucial in the stabilization and control of these vehicles. To cite two examples, consider the stability of a dual-spin spacecraft with flexible appendages or the potential instabilities in the active control of flexible spacecraft.

Modern structural analysis techniques have tended to focus on the "stiffness matrix" $\boldsymbol{K}$ and the "system inertia matrix" $\boldsymbol{M}$, thereby generating structural-dynamical models of the form

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{q}$ contains suitable generalized coordinates, and $\boldsymbol{f}$ contains the generalized inputs. A notable omission in Eq. (1) is a mechanism for energy dissipation. A common device to remedy this omission is the addition of a linear-viscous damping term $\boldsymbol{D}\dot{\boldsymbol{q}}$. Hence, the "improved" system is

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f}(t) \tag{2}$$

Very little is said in the literature concerning how to calculate $\boldsymbol{D}$. This is not surprising because none of the expected forms of dissipation (including material damping, structural damping, hysteretic damping, stiction, Coulomb damping, freeplay at bolts, rivets, or joints) is in fact linear-viscous damping.

The next step is often to find the system modes for Eq. (2). This requires the simultaneous diagonalization of $\boldsymbol{M}$, $\boldsymbol{D}$, and $\boldsymbol{K}$, which is not, in general, possible. As a mathematical curiosity, this triple diagonalization is possible if $\boldsymbol{D}$ is a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$ (this condition is sufficient, but not necessary). However, the author knows of no physical justification for this assumption. Denoting by $\boldsymbol{T}$ the transformation that diagonalizes $\boldsymbol{M}$ and $\boldsymbol{K}$, so that

$$\boldsymbol{T}^T\boldsymbol{M}\boldsymbol{T} = \boldsymbol{1} \qquad \boldsymbol{T}^T\boldsymbol{K}\boldsymbol{T} = \boldsymbol{\omega}^2 \tag{3}$$

where $\boldsymbol{1}$ is the unit matrix, and $\boldsymbol{\omega}$ is a diagonal matrix of (undamped) natural frequencies $\omega_\alpha$, the change of variables

$$\boldsymbol{q} = \boldsymbol{T}\boldsymbol{\eta} \tag{4}$$

converts Eq. (2) to

$$\ddot{\boldsymbol{\eta}} + \hat{\boldsymbol{D}}\dot{\boldsymbol{\eta}} + \boldsymbol{\omega}^2\boldsymbol{\eta} = \hat{\boldsymbol{f}}(t) \tag{5}$$

where $\hat{\boldsymbol{f}} \triangleq \boldsymbol{T}^T\boldsymbol{f}$ and $\hat{\boldsymbol{D}} \triangleq \boldsymbol{T}^T\boldsymbol{D}\boldsymbol{T}$, with $\hat{\boldsymbol{D}}$ generally having off-diagonal elements. In the crusade for uncoupled modal equations, the off-diagonal terms $\hat{d}_{\alpha\beta}$ $(\alpha \neq \beta)$ are often set to zero.


Received Nov. 19, 1979; revision received Feb. 19, 1980.



Index categories: Spacecraft Dynamics and Control; Structural Dynamics.


*Professor, Institute for Aerospace Studies. Associate Fellow AIAA.